THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2070A (First Term, 2019-20)
Algebraic Structures
Course Outline

Outline

This course is intended as an introduction to modern abstract algebra and the algebraic way of thinking in advanced mathematics. The course focuses on basic algebraic concepts which arise in various areas of advanced mathematics, and emphasizes on the underlying algebraic structures which are common to various concrete mathematical examples.

Topics include:

- Group Theory - examples of groups including permutation and dihedral groups, subgroups, the Theorem of Lagrange, group homomorphisms.
- Ring Theory - examples of rings including the ring of integers and polynomial rings, integral domains, fields, ring homomorphisms, ideals and quotient rings.
- Field Theory - examples of field extensions and finite fields.

Prerequisites

Students taking this course should have taken MATH 1010 *University Mathematics* and MATH 1050 *Foundation of Modern Mathematics* (or classes at equivalent level). It would be very helpful if you have also taken MATH 1030 *Linear Algebra I*.

Class Information

- Instructor: CHAN Kwok Wai (Office: LSB 212; Email: `kwchan@math.cuhk.edu.hk`)
- Teaching Assistant:
  - NG Ming Ho (Office: LSB 228; Email: `mhng@math.cuhk.edu.hk`)
  - WANG Dan (Office: AB1 505; Email: `dwang@math.cuhk.edu.hk`)
- Lectures: Mon 2:30pm - 3:15pm at LSB LT4; Wed 2:30pm - 4:15pm at LSB C2
- Tutorials: Mon 3:30pm - 4:15pm at LSB LT4
- Webpage: `https://www.math.cuhk.edu.hk/course/1920/math2070a`

Suggested Texts

- Lecture notes available at the course webpage.
- Artin, *Algebra*, Prentice Hall, 2nd edition.
- Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley, 7th edition (textbook for MATH 3030).

Lectures, Tutorials and Homeworks

Lectures: The lectures will focus mainly on the theoretical concepts and proofs, supplemented occasionally with some illustrative examples. The chapters in the lecture notes, which are posted online already, are numbered by weeks. As the lectures would only cover the most essential materials (at a rather fast pace), it would be very helpful if you have read (or at least skimmed through) the relevant chapter beforehand.

Tutorials: The tutorials will cover more concrete examples and computational aspects of the materials. The style is more informal and students are encouraged to discuss and work out some exercises together. All the materials (except otherwise stated) covered in lectures and tutorials will be covered in the midterm and final exam.

Homework assignments: There will be weekly problem sets, usually posted on Mondays and due on Thursday in the following week. Each problem set consists of two parts – the compulsory part and the optional part. You only need to hand in your solutions of the compulsory part. But you are highly recommended to work out the optional part at home as well. The full mark for each homework is 1 point. There will be around 10 sets of homework assignments but at most the best 8 of your solutions will be counted. You may also choose to hand in $0 \leq n \leq 8$ solution sets and let your Final constitute $(58 - n)\%$ of the total score.

Assessment

- n%: Homework (where $0 \leq n \leq 8$)
- 42%: Midterm (28^{th} Oct 2019, Monday, 2:30pm in class)
- (58 - n)%: Final

Tentative Schedule

- Week 1–5: Group theory
  - definition and basic examples of groups
  - cyclic groups, symmetric groups and dihedral groups
  - subgroups, cyclic subgroups and generating sets
  - equivalence relations and partitions; cosets and the Theorem of Lagrange
  - group homomorphisms and isomorphisms, and some basic examples
- Week 6–12: Ring theory
  - definition and basic examples of rings
  - polynomials and polynomial rings
  - integral domains and fields; field of fractions
  - ring homomorphisms and isomorphisms, and some basic examples; subrings and ideals
  - quotient rings
  - factorization of polynomials, Euclidean algorithm, gcd
  - Gauss' Lemma and Eisenstein's Criterion
- Week 13: Field theory
  - basic examples of field extensions and finite fields